

Let us consider the collision of two bodies, the law of conservation of momentum states that in the absence of external forces sum of momentum before collision is equal to sum of momenta of the bodies considered, after collision. If p_1 and p_2 be of the momenta, then according to the law of conservation of momentum.

Momentum before Collision = Momentum after Collision

$$(p_1 + p_2)_{\text{before}} = (p_1 + p_2)_{\text{after}}$$

where p is momentum defined as

$$p = mv$$

Derivation : —

Let us consider that the collision takes place in a region of space which is far away from external forces. We also assume that the law of conservation of energy holds.

Let there be two free particles having velocities u_1 and u_2 in the initial position. These two particle collide and after collision they have velocities v_1 and v_2 . Then if m_1 and m_2 are the masses of the particles considered, the initial kinetic energy of the two particles is

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 =$$

After collision the kinetic energy is

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

According to the law of conservation of energy

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \Delta W$$

where ΔW is the change in excitation energy of the particles, consequent to the collision. This energy may be internal vibrational or rotational energy. In an elastic collision $\Delta W = 0$. The law of conservation of momentum holds good whether the collision is elastic or inelastic. Now if we observe the same collision from the system s' moving with respect to s with velocity v , then according to Galilean transformation with velocity, we have

$$\left. \begin{aligned} u'_1 &= u_1 - v \\ u'_2 &= u_2 - v \\ v'_1 &= v_1 - v \\ v'_2 &= v_2 - v \end{aligned} \right\}$$

According to law of conservation of energy in two systems, we have energy before collision + ΔW , ΔW is the external excitation energy which is constant in both the frames

$$\frac{1}{2} m_1 u_1'^2 + \frac{1}{2} m_2 u_2'^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 + \Delta W \quad (7)$$

where u_1' , u_2' , v_1' and v_2' represent the corresponding velocities in system S'.

Substituting values from eqn (6), we get

$$\frac{1}{2} m_1 (u_1 - v)^2 + \frac{1}{2} m_2 (u_2 - v)^2 = \frac{1}{2} m_1 (v_1 - v)^2 + \frac{1}{2} m_2 (v_2 - v)^2 + \Delta W$$

$$\begin{aligned} \text{or } & \frac{1}{2} m_1 (u_1^2 + v^2 - 2u_1v) + \frac{1}{2} m_2 (u_2^2 + v^2 - 2u_2v) \\ & = \frac{1}{2} m_1 (v_1^2 + v^2 - 2v_1v) + \frac{1}{2} m_2 (v_2^2 + v^2 - 2v_2v) \\ & \quad + \Delta W \end{aligned}$$

$$\begin{aligned} \text{or } & \frac{1}{2} m_1 u_1^2 - m_1 u_1 v + \frac{1}{2} m_2 v_2^2 - m_2 u_2 v \\ & = \frac{1}{2} m_1 v_1^2 - m_1 v_1 v + \frac{1}{2} m_2 v_2^2 - m_2 v_2 v + \Delta W \end{aligned}$$

Since the terms containing v^2 cancel from both sides

$$(m_1 u_1 + m_2 u_2)v = (m_1 v_1 + m_2 v_2)v \quad (8)$$

from equation (5)

This equation holds for all value of v

Therefore we have

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\text{or } (\beta_1 + \beta_2) \underset{\text{Before}}{=} (\beta_1 + \beta_2) \underset{\text{After}}{=}$$

Before After

Thus the law of conservation of momentum holds under Galilean transformation.

Example:- The position vectors of the two velocities particles at any instant are \vec{r}_1 and \vec{r}_2 respectively and their velocity vectors are \vec{v}_1 and \vec{v}_2 respectively.

Prove that they can collide if and only if the vectors $(\vec{r}_1 - \vec{r}_2)$ and $(\vec{v}_1 - \vec{v}_2)$ are in the same direction.

Solution:- Let there be two particles P and Q whose position vectors \vec{r}_1 and \vec{r}_2 relative to origin. The velocity vectors of P and Q are

\vec{v}_1 and \vec{v}_2 at that instant as shown in figure.

After time interval t , let the two particles collide at point C.

At point C the position vector particle P

$$= \overrightarrow{OC} = \overrightarrow{OF} + \overrightarrow{FC} = \vec{r}_1 + \vec{v}_1 t$$

$$\text{At point Q} \quad = \overrightarrow{OC} = \overrightarrow{OQ} + \overrightarrow{QC} = \vec{r}_2 + \vec{v}_2 t$$

The particle P and Q can only collide. At the instant of collision $\vec{r}_1 + \vec{v}_1 t = \vec{r}_2 + \vec{v}_2 t$

$$\text{or } (\vec{r}_1 - \vec{r}_2) = (\vec{v}_2 - \vec{v}_1)t$$

This mean $\vec{r}_1 - \vec{r}_2$ and $\vec{v}_2 - \vec{v}_1$ are parallel hence

$$(\vec{r}_1 - \vec{r}_2) \times (\vec{v}_2 - \vec{v}_1) = 0$$